Coulomb drag by small momentum transfer between quantum wires

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We demonstrate that in a wide range of temperatures Coulomb drag between two weakly coupled quantum wires is dominated by processes with a small interwire momentum transfer. Such processes, not accounted for in the conventional Luttinger liquid theory, cause drag only because the electron dispersion relation is not linear. The corresponding contribution to the drag resistance scales with temperature as T^2 if the wires are identical, and as T^5 if the wires are different.

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Electrons moving in a conductor generate a fluctuating electric field around it. This field gives rise to an unusual transport phenomenon, Coulomb drag between two closely situated conductors [1]. The structure of the fluctuating field is determined by electron correlations within the conductors. Correlations are stronger in conductors of lower dimensionality. Tomonaga-Luttinger model captures some aspects of the correlations in the case of one-dimensional conductors (quantum wires). Within this model, the Coulomb drag was studied in [2, 3]. In a typical setup [4, 5] a dc current flows through the active wire 1, while the bias applied to the passive wire sets $I_2 = 0$, see Fig. 1. The *drag resistivity* (drag resistance per unit length of the interacting region) is then defined as

$$r = -\lim_{I_1\to 0} \frac{e^2}{2\pi\hbar}\frac{1}{L}\frac{dV_2}{dI_1}. \tag{1}$$

The only source of drag in the Luttinger liquid is interwire backscattering, associated with a large momentum transfer between the wires. The model predicts a distinctive temperature dependence of the corresponding contribution r_{2k_F} to the drag resistivity (1). In the case of identical wires $r_{2k_F} \propto l_{2k_F}^{-1} e^{\Delta/T}$ at the lowest temperatures [2, 3]. Here l_{2k_F} is the scattering length characterizing the interwire backscattering. At temperatures T above the gap Δ, this exponential dependence is replaced by a power-law, $r_{2k_F} \sim l_{2k_F}^{-1}(T/\epsilon_F)^{1-\gamma}$, where ϵ_F is the Fermi energy. The exponential temperature dependence of r_{2k_F} indicates a formation of a zig-zag charge order due to the $2k_F$-component of the interwire interaction [2, 3]. To the contrary, the exponent $\gamma > 0$ in the power-law portion of the function $r_{2k_F}(T)$ is determined by the interactions within the wires; $\gamma = 0$ in the absence of interactions [6]. This renormalization of r_{2k_F} is similar in origin to the suppression of the conductance of a Luttinger liquid with an impurity [7]: in both cases repulsive interactions enhance the backscattering probability when temperature is lowered.

However, *forward* scattering between the wires also induces drag. To see this, one has to go beyond the Tomonaga-Luttinger model and account for the nonlinearity of the electronic dispersion relation. If the electron velocity depends on momentum, then even small (compared to $2k_F$) momentum transfer results in drag.

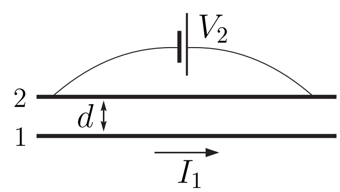

FIG. 1: Coulomb drag between quantum wires. A dc current I_1 flows through the active wire (1). A voltage bias V_2 is applied to the passive wire (2) in such a way that $I_2 = 0$.

The small (r_0) and large (r_{2k_F}) momentum transfer contributions to the drag are inversely proportional to the corresponding scattering lengths l_0 and l_{2k_F}, respectively. Their ratio $l_0/l_{2k_F} \propto e^{-4k_F d}$ depends strongly on the distance d between the wires. If the drag effect is employed to study the correlations within a wire, rather than the zig-zag order induced by interwire interaction, then d must be large: $k_F d > 1$. In this case the gap $\Delta \sim \epsilon_F (k_F l_{2k_F})^{-1/\gamma}$ becomes narrow, and the role of forward scattering increases.

In this paper we demonstrate that the drag resistivity between weakly coupled wires is dominated by the forward scattering in a wide temperature range. Even for identical wires, which is the most favorable for backscattering case, $r_0 \propto T^2$ wins over r_{2k_F} at all T above $T^* \sim \epsilon_F (l_0/l_{2k_F})^{1/(1+\gamma)}$. For different wires, r_{2k_F} is exponentially small at $T \lesssim u\delta n$, whereas r_0 has a power-law low-temperature asymptotics; here δn is the mismatch of the electron densities between the wires and u is the characteristic plasma velocity; hereafter we set $\hbar = 1$.

The Hamiltonian of the system, $H = H_1 + H_2 + H_{12}$, is given by the sum of the Hamiltonians of the two isolated wires $i = 1, 2$,

$$H_i = \int dx\, \psi_i^\dagger(x) \frac{\hat{p}^2}{2m} \psi_i(x) + H_{int}, \quad \hat{p} = -id/dx,$$

$$H_{int} = \int dxdy\, \rho_i(x) U_{ii}(x-y) \rho_i(y), \tag{2}$$

and of the Hamiltonian of the interwire interaction,

$$H_{12} = \int dx_1 dx_2\, \rho_1(x_1) U_{12}(x_1 - x_2) \rho_2(x_2); \qquad (3)$$

here $\rho_i(x) = \psi_i^\dagger(x)\psi_i(x)$.

We will assume that the interactions are screened by the nearby metallic gates. The screening length d_s is set by the distance to the gates and is typically [4, 5] of the order of the separation d between the wires, $d_s \sim d$. The short-distance cutoff d_{ij} of the Coulomb potential is provided by the wire width d_0 for $i = j$ or by the interwire distance d for $i \neq j$. The Fourier transforms $U_{ij}(k) = \int dx e^{ikx} U_{ij}(x)$ are rapidly decreasing functions of k with $U_{ij}(k) \propto e^{-|k|d_s}$ for $|k| \gg 1/d_s$, and $U_{ij}(k) \approx \text{const}$ for $|k| \ll 1/d_{ij}$. Note that $d_{ij} \sim d_s \sim d$ for interwire interaction $U_{12}(k)$. Thus, its k-dependence is characterized by a single scale $k_0 \sim 1/d$.

Because of the interaction H_{12}, electrons in the wire 2 experience a force [2] whose density is given by

$$\mathcal{F}_2 = \int dx\, (dU_{12}(x)/dx)\, \langle \rho_1(x)\rho_2(0)\rangle. \qquad (4)$$

Since there is no current in the wire 2, this force must be balanced by an external electric field, $en_2\mathcal{E}_2 + \mathcal{F}_2 = 0$, where $\mathcal{E}_2 = V_2/L$ and $n_i = \langle \rho_i \rangle$ is the concentration of electrons in the wire i. At $T \gg \Delta$ (see the discussion above) the correlation function in the r.h.s. of Eq. (4) can be evaluated in the first order in U_{12},

$$\frac{V_2}{L} = \frac{1}{en_2} \int \frac{dk d\omega}{(2\pi)^2}\, k U_{12}^2(k) \widetilde{S}_1(k,\omega) \widetilde{S}_2(-k,-\omega), \qquad (5)$$

where $\widetilde{S}_i(k,\omega)$ are the dynamic structure factors,

$$\widetilde{S}_i(k,\omega) = \int dx dt\; e^{i\omega t - ikx} \langle \rho_i(x,t)\rho_i(0,0)\rangle,$$

calculated in the presence of a finite current I_1 in the wire 1. The structure factor $\widetilde{S}_2(k,\omega)$ in the wire 2 coincides with its equilibrium value, $S_2(k,\omega)$. The electronic subsystem in the wire 1 is in equilibrium in the reference frame moving with the drift velocity $v_d = I_1/en_1$ in the direction of the current. Therefore the structure factor $\widetilde{S}_1$ is obtained from the equilibrium value S_1 using the Galilean transformation: $\widetilde{S}_1(k,\omega) = S_1(k, \omega - qv_d)$. Equations (1) and (5) then yield

$$r = \int dk d\omega\; \frac{k^2 U_{12}^2(k)}{8\pi^3 n_1 n_2} \frac{\partial S_1(k,\omega)}{\partial \omega} S_2(-k,-\omega). \qquad (6)$$

Now we use the fluctuation-dissipation theorem,

$$S_i(k,\omega) = \frac{2A_i(k,\omega)}{1 - e^{-\omega/T}}$$

to further simplify Eq. (6),

$$r = \int_0^\infty dk \int_0^\infty d\omega\; \frac{k^2 U_{12}^2(k)}{4\pi^3 n_1 n_2 T} \frac{A_1(k,\omega) A_2(k,\omega)}{\sinh^2(\omega/2T)}. \qquad (7)$$

Here A_i is the imaginary part of the retarded density-density correlation function; $A_i(k,\omega) = A_i(-k,\omega) = -A_i(k,-\omega)$. Equation (7) was derived by different means in [8]; similar expressions have been also obtained for noninteracting systems with disorder [9]. Here we demonstrated the validity of Eq. (7) for clean interacting systems.

We start with the evaluation of the drag resistivity for noninteracting electrons ($U_{ii} = 0$). Concentrating on the small momentum transfer contribution to r, we consider the limit $l_0/l_{2k_F} \to 0$, thus setting $r_{2k_F} = 0$. In this case the main contribution to the integral over k in Eq. (7) comes from small momenta $k \ll k_F$ and small energies $\omega \ll \epsilon_F$. At these values of k and ω, functions $A_i(k,\omega)$ are sharply peaked at $\omega_i = v_i k$, where $v_i = \pi n_i/m$ are the Fermi velocities in the two wires. For a given $k < 2k_F$ the widths of the peaks can be estimated as

$$\delta\omega(k,T) = \max\left\{k^2/m, kT/k_F\right\}. \qquad (8)$$

Equation (8) and the exact f-sum rule,

$$\int_0^\infty d\omega\; \omega A_i(k,\omega) = \frac{\pi n_i}{m}\frac{k^2}{2}, \qquad (9)$$

allow us to estimate the peak heights: $A_i \sim k/2\delta\omega$. If the difference between the Fermi velocities is small,

$$\delta v = |v_1 - v_2| \ll v_F = \pi n/m, \quad n = (n_1 + n_2)/2,$$

then Eq. (7) reduces to

$$r = \frac{1}{4\pi^3 n^2 T} \int_0^\infty dk\; \frac{k^2 U_{12}^2(k)}{\sinh^2(v_F k/2T)}\, \alpha(k,T), \qquad (10)$$

$$\alpha(k,T) = \int_0^\infty d\omega A_1(k,\omega) A_2(k,\omega). \qquad (11)$$

The function $\alpha(k,T)$ depends on δv. If the wires are identical ($\delta v = 0$), then Eq. (11) and the above estimates for A_i yield

$$\alpha(k,T) \approx \frac{k^2}{4\delta\omega(k,T)}. \qquad (12)$$

There are two competing scales in the integrand of Eq. (10). The first scale, $k_0 \sim 1/d \ll k_F$, characterizes the k-dependence of the interwire interaction $U_{12}(k)$. The typical wave vector of thermally excited electron-hole pairs, T/v_F, defines the second scale. The two scales coincide at $T = T_0 = v_F k_0$. At $T \ll T_0$ one can replace $U_{12}(k)$ by $U_{12}(0)$ in Eq. (10). Furthemore, we use α in the form of Eq. (12) at $T = 0$, which results in

$$r = \frac{c_1}{l_0}\left(\frac{T}{\epsilon_F}\right)^2, \quad \frac{1}{l_0} = \left[\frac{U_{12}(0)}{2\pi v_F}\right]^2 n \qquad (13)$$

with $c_1 = \pi^4/12$. Use of exact form of $A_i(k,\omega)$ in Eq. (11) changes only the numerical coefficient, $c_1 = \pi^2/4$.

The increase of temperature T above T_0 results in a saturation of the drag resistivity. Indeed, at $T_0 \ll T \ll \epsilon_F$ one can expand $\sinh(v_F k/2T)$ in Eq. (10) and use $\delta\omega = kT/k_F$ for the peak width in (12). This yields

$$r \sim \frac{1}{l_0}\int_0^\infty \frac{kdk}{n^2}\frac{U_{12}^2(k)}{U_{12}^2(0)} \sim \frac{1}{l_0}\left(\frac{T_0}{\epsilon_F}\right)^2. \qquad (14)$$

Further increase of T leads to the decay of the drag,

$$r \propto l_0^{-1}(T_0/\epsilon_F)^2(T/\epsilon_F)^{-3/2}, \quad T \gg \epsilon_F, \qquad (15)$$

similar to the two-dimensional case [10].

We now consider wires with slightly different Fermi velocities $\delta v > 0$. In this case the peaks of $A_i(k,\omega)$ are separated in ω by $k\delta v$. We define a new temperature scale $T_1 = k_F\delta v$ by equating the separation to the peak width (8). We assume this scale is small, $T_1 \ll T_0$. The difference between velocities does not affect the drag at $T \gg T_1$. However, at $T \ll T_1$ the drag resistivity is suppressed exponentially. To obtain the leading asymptotics of $r(T)$ it is sufficient to use the $T = 0$ limit [11] of $A_i(k,\omega)$ in Eq. (11), $\alpha(k,0) = (m/4k)\,(k - m\delta v)\,\theta(k - m\delta v)$. Equation (10) then results in

$$r = \frac{\pi^2/4}{l_0}\left(\frac{T_1}{\epsilon_F}\right)^2 \frac{T}{T_1}e^{-T_1/T}, \qquad T \ll T_1. \qquad (16)$$

The activational temperature dependence Eq. (16) holds for all $T \ll T_1$, because for noninteracting electrons at $T = 0$ the product $A_1(k,\omega)A_2(k,\omega)$ is exactly zero [11] at $k < m\delta v$. If electrons interact, some overlap of A_1 and A_2 exists even at small $k \ll m\delta v$. This yields a further contribution to r, that has a power-law temperature dependence. We will evaluate this contribution for weak intrawire interaction.

It is convenient to write $A(k,\omega)$ (we suppress the index i in the following) in the form $A(k,\omega) = [S(k,\omega) - S(-k,-\omega)]/2$ and use the Lehmann representation for the dynamic structure factor:

$$S(k,\omega) = \frac{2\pi}{L}\sum_n |\langle n|\,\rho_k\,|gs\rangle|^2\,\delta(\omega - E_n + E_{gs}). \qquad (17)$$

Here L is the system size, $|gs\rangle$ is the ground state, and $\rho_k = \sum_p \psi^\dagger_{p+k}\psi_p$. We evaluate the matrix element in Eq. (17) in the first order in the intrawire interaction. The nonvanishing at $\omega - v_F k \gg \delta\omega$ contribution results from the processes in which the unperturbed final state $|n\rangle$ in Eq. (17) has two electron-hole pairs: $|n\rangle^{(0)} = \psi^\dagger_{p+q}\psi_p\psi^\dagger_{p'-q'}\psi_{p'}\,|0\rangle$. This contribution is

$$\begin{aligned}\delta S(k,\omega) = \frac{1}{\pi^2}\int dp dp' dq dq'\,\delta(q - q' - k) \qquad (18)\\ \times\,\delta(\omega - \xi_{p+q} + \xi_p - \xi_{p'-q'} + \xi_{p'})\\ \times\, f_p(1 - f_{p+q})f_{p'}(1 - f_{p'-q'})K^2(p,p',q,q',\omega),\end{aligned}$$

where f_p are the Fermi functions, $\xi_p = p^2/2m$, and

$$\begin{aligned}K \;=\; &\frac{U(q') - U(p - p' + q')}{\omega - \xi_{p+q} + \xi_{p+q'}} - \frac{U(q') - U(p - p' + q)}{\omega - \xi_{p+q-q'} + \xi_p}\\ &+[p \leftrightarrow p', q \leftrightarrow -q'].\end{aligned}$$

Note that Eq. (18), unlike Eq. (17), accounts for a finite temperature. At $\omega \ll \epsilon_F$ and $k \ll k_F$, Eq. (18) yields the interaction-induced correction to $A(k,\omega)$,

$$\delta A(k,\omega) = \frac{\widetilde{U}^2}{v_F}\frac{k^4}{m^2}\frac{\theta(\omega - v_F k)}{\omega^2 - v_F^2 k^2}, \qquad (19)$$

where $\widetilde{U} = [U(0) - U(2k_F)]/2\pi v_F \ll 1$. This result is valid for $\omega \ll \epsilon_F$, $k \ll k_F$, $|\omega - v_F k| \gg \max\{\widetilde{U}v_F k, \delta\omega(k,T)\}$, and describes $A(k,\omega)$ outside the interval Eq. (8). The limit of linear electron dispersion relation ($m \to \infty$) corresponds [12] to $\delta A(k,\omega) = 0$.

We use Eq. (19) to evaluate the interaction-induced correction δr to the drag resisitivity between non-identical wires with $T_1 = k_F\delta v \gg \epsilon_F\widetilde{U}$. At the lowest temperatures, Eqs. (10) and (11) yield

$$\delta r \sim \frac{\widetilde{U}^2}{l_0}\left(\frac{T_1}{\epsilon_F}\right)^4\left(\frac{T}{T_1}\right)^5. \qquad (20)$$

With increasing temperature, the $r(T)$ dependence changes from Eq. (20) to the activation law (16). At $T \gg T_1$ the difference between wires does not affect $r(T)$.

We will argue now that intrawire interactions do not change the quadratic temperature dependence of $r(T)$ at $T_1 \ll T \ll T_0$, see Eq. (13). At these temperatures, an estimate equivalent to Eq. (13) reads $r \sim |U_{12}^2(0)|v_F^{-3}\delta\omega(k_T,T)$ and yields $r \propto T^2$; here $k_T \sim T/v_F \ll k_F$ is the wave vector of a typical electron-hole excitation. Interaction apparently does not affect the functional dependence of $\delta\omega$ on k and T; the estimate (8) still can be used, although the coefficients $1/m$ and $1/k_F$ in it are affected by the interaction.

The Tomonaga-Luttinger model is insufficient for the evaluation of $\delta\omega$ in the presence of interaction: it implies linear electron spectrum, which yields [12] $\delta\omega = 0$. Accounting for the curvature of the electron spectrum complicates the treatment of the interaction greatly. The width $\delta\omega$ can be explicitly evaluated in the Calogero-Sutherland model which is characterized by a very specific interaction potential,

$$U_{ii}(x) = \frac{2\pi^2}{mL^2}\frac{\lambda(\lambda-1)}{\sin^2[\pi x/L]}. \qquad (21)$$

The parameter λ here is related to the conventional interaction parameter g of the Luttinger liquid: $g = 1/\lambda$. This relation follows from the definition $g = v_F/u$ in terms of the velocity of the collective mode (plasmon) u, and its value $u = (\pi n/m)\lambda$ in the Calogero-Sutherland model [13, 14]. For the rational values of λ and at

$T = 0$ the density-density correlation function is known exactly [13, 14]. Due to the integrability of the model, $A_i(k,\omega) \neq 0$ only in a finite interval of ω around $\omega = u_i k$ [15]. We found this interval for $k \leq 2\pi n_i$:

$$-(1/g)\frac{k^2}{2m} < \omega - u_i k < \frac{k^2}{2m}, \qquad (22)$$

which yields for the width

$$\delta\omega(k,0) = \frac{1+g}{2g}\frac{k^2}{m}. \qquad (23)$$

In order to estimate r we note that Eq. (7) and the sum rule (9) remain valid in the presence of interactions within the wires. This allows us to follow the steps that led to Eq. (13). Replacing v_F by the plasma velocity u in Eq. (10) and using Eq. (23), we find

$$r = \frac{c_g}{l_0}\left(\frac{T}{\epsilon_F}\right)^2, \quad c_g \propto \frac{g^6}{1+g}, \qquad (24)$$

which agrees with our expectation for the $r(T)$ dependence. We are not aware of a reliabale theory of $A_i(k,\omega)$ beyond the exactly solvable case. However, the self-consistent Born approximation results [16] allow us to corroborate the estimate $\delta\omega \propto k^2/m$ for the peak width, so, apparently, the $r \propto T^2$ dependence is universal.

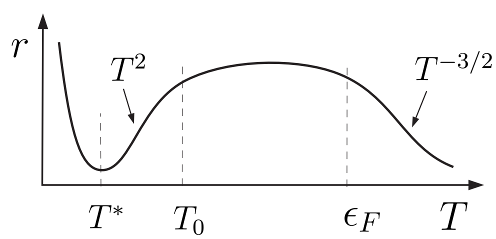

FIG. 2: Sketch of the temperature dependence of the drag resistivity between identical wires. The small momentum transfer contribution considered in this paper dominates at $T > T^*$; the ratio T^*/ϵ_F is exponentially small for $k_F d > 1$.

First observations of drag between ballistic quantum wires appeared recently [4, 5]. In a limited temperature interval, $0.2\text{ K} < T < 0.9\text{ K}$, a three-fold drop in the drag resistance was observed [4] with the increase of temperature. This drop was fit to a power-law $r \propto T^{-0.77}$ and interpeted as evidence of the Luttinger liquid behavior. However, the Fermi wave vector in the wires of Ref. [4] was estimated to be $k_F = 6 \times 10^4\ \text{cm}^{-1}$, which yields $\epsilon_F = \hbar^2 k_F^2/2m^* \approx 0.2$ K (we used here $m^* = 0.068\, m_0$ known for GaAs). It thus appears that the measurements of Ref. [4] correspond to a non-degenerate or weakly degenerate regime incompatible with the Luttinger liquid description. An alternative explanation of the observations [4, 5] is provided by our theory. Indeed, using the values of k_F and $d = 200$ nm of [4], we find $k_F d = 1.2$. Under this condition the small momentum transfer contribution dominates at $T > T^*$, see Fig. 2. The observed [4, 5] behavior of $r(T)$ may correspond to the crossover regime between the limits $r(T) = \text{const}$ and $r(T) \propto T^{-3/2}$ presented by Eqs. (14) and (15).

To conclude, the small momentum transfer contribution dominates Coulomb drag at almost all temperatures if the distance between the wires exceeds the Fermi wavelength, see Fig. 2. Drag by small momentum transfer is possible because electron dispersion relation is not linear, and therefore can not be accounted for in the conventional Tomonaga-Luttinger model.

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